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PROBLEM-SOLVING ABILITY IN REALISTIC MATHEMATICS EDUCATION BASED ON HYPOTHETICAL LEARNING TRAJECTORY

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Abstract: Problem-solving ability is one of the essential learning characteristics in 21st-century skills, which requires students to think critically during the problem-solving process. Thus, the purpose of this study was to describe students' problem-solving skills in HLT-based RME learning. This research used a qualitative method with a case study design. The research was implemented in SMP Negeri 3 Satu Atap Tawangharjo. The research subjects were the principal, math subject teacher, and seventh-grade students. Data validity used triangulation of methods and sources. Data analysis technique with flow method. The results showed that some students had high, medium, and low mathematical problem-solving abilities in solving math problems. In conclusion, students' problem-solving ability in HLT-based RME learning has a high percentage. The percentage of students with high problem-solving ability is 75%, those with moderate problem-solving ability make up 15%, and those with low problem-solving ability account for 10%. Thus, students who were able to solve problems in HLT-based RME learning were more than students who were not able to solve problems in HLT-based RME learning.

Keywords: problem-solving ability; hypothetical learning trajectory; realistic mathematics education

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Introduction

The independent curriculum that is now the basis of Indonesian education implements learning that leads to 21st-century learning. 21st-century learning refers to an approach that prepares students with essential 21st-century skills, known as the 4Cs: communication, collaboration, critical thinking and problem-solving, as well as creativity and innovation (Aji, 2019). Mastery of 21st-century skills is characterised by the need for students to develop critical thinking skills to solve problems. According to Fitria et al., (2020), in the independent curriculum, the application of 21st-century learning has a positive effect so that the next generation of this country can face problems in the 21st century. To find suitable solutions, students must have mathematical problem-solving skills (Octaviana & Setyaningsih, 2022). However, Indonesian students' ability to solve mathematical problems could be higher compared

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to other countries (Safitri et al., 2020). This is reflected in the Program for International Student Assessment (PISA) results in 2018, where Indonesia was ranked 73 out of 79 countries. Therefore, efforts are needed to improve students' ability to solve math problems to solve problems well (Wandanu et al., 2020).

Learning mathematics is not just students' mastery of mathematical knowledge, but understanding and mastery of problem-solving skills is also the key to achieving learning. Mathematical problem-solving skills are essential for students, requiring them to engage actively in various activities like exploration, experimentation, observation, and investigation (Gee, 2019b; Putra et al., 2018; Putri Solihat et al., 2022; D. R. Sari & Fauzan, 2024). Thus, problem-solving skills are critical to overcoming everyday mathematics-related problems (Zahro & Haerudin, 2022). Polya's steps are the target of several methods that can be used to develop problem-solving skills. According to Polya (2014), four indicators of problem-solving ability, namely: (1) understanding the problem, (2) planning the solution, (3) implementing the solution plan, (4) re-examining the solution. Problem-solving activities with Polya's steps are an effort to increase students' level of understanding and be able to determine the steps of solving (Setiana et al., 2021). The lack of students' mathematical problem-solving skills also impacts the low quality of mathematics learning in Indonesia (Khairunnisa & Rakhman, 2023). The significance of selecting an appropriate learning model lies in a teacher's effort to enhance the quality of education (Fadhilla & Afri, 2024). Some mathematics teachers said that the primary goal of the teaching and learning process is to improve students' problem-solving skills. They added that interaction between teachers, students, and other elements can be achieved to achieve this goal (Lester, 2013). Thus, the learning model used by the teacher must match the daily problems of students concretely to make it easier to solve the problems faced.

One of the learning models that is used by students' daily problems is Realistic Mathematics Education (RME) based on the Hypothetical Learning Trajectory (HLT) (Febriani & Sidik, 2020). RME is the foundation of Hans Freudental's interpretation of mathematics into an activity that focuses on student concept discovery and develops mathematical knowledge with mathematical problems presented in actual form (Abrika et al., 2023; Hidayati et al., 2022; D. R. Sari & Fauzan, 2024). The principle of RME learning activities is that students find the concept of material from contextual problems provided by the teacher (Freudenthal, 1986). Implementing RME can improve critical reasoning skills in students' fraction problem-solving abilities (Aprilianto & Sutarni, 2023). The essence of RME learning is that teachers are expected to organise activities that support the concept discovery process, so that students can develop their understanding through the activities they undergo during learning (Elwijaya et al., 2020). The development of RME learning positively impacts students' problem-solving skills (Fauzan et al., 2018; Program et al., 2023; D. R. Sari & Fauzan, 2024). The implementation of RME learning needs a learning process plan that is used to guide teachers and students. HLT, as one of the appropriate designs for the mathematics learning process, is equipped with teacher and student activities (Hidayati et al., 2022).

HLT was first used by Simon (2020), who designed HLT by including objectives, activities, and opportunities for events that arise in the learning process. HLT is the basis for designing activities that will be carried out in the teaching and learning process to achieve the predetermined learning objectives (E. A. P. Sari, 2011). In addition, using HLT involves various activities that aim to improve students' thinking ability when structuring the material. Thus, HLT helps students rediscover mathematical ideas and train their ability to construct their thinking when solving problems (Gee, 2019). The findings of Wandanu et al., (2020) demonstrated that HLT based on RME can enhance students' mathematical problem-solving skills. Furthermore, Gee (2019) research revealed an improvement in students' mathematical problem-solving abilities after participating in RME-based learning. This is evident from the average problem-solving skill score before the intervention, which fell into the very low category at 48.41. Meanwhile, after the action, the average value increased to the good category, reaching 74.85.

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Then, the research results by Hidayati et al. (2022), the application of RME-based HLT improves mathematical problem-solving skills on SPLDV material. However, further research is needed on problem-solving abilities in RME-based HLT learning, specifically on the topic of fractions. Likewise, in State Junior High School 3 One Roof Tawangharjo still needs to improve in mathematical problem-solving.

Based on this description, this article's research objectives are to describe students' problem-solving skills in HLT-based RME learning.

Method

This type of research uses qualitative research with a case study design. Qualitative research is called a naturalistic method because the research is conducted in natural conditions, environments, or settings (Sutama et al., 2022). This research aims to describe students' problem-solving abilities in HLT-based RME learning. The subjects of this study consisted of the Principal, Mathematics Teacher, and 20 students of class VII of State Junior High School 3 One Roof Tawangharjo who had studied fraction material.

Data collection techniques were observation, tests, interviews, and document analysis. Tests were used to measure students' problem-solving skills. Observation is done to determine how students receive learning concepts and their application in daily life problems (Baskoro et al., 2013). Document analysis was conducted by collecting answer sheets, interview results, observation sheet results, and recordings or pictures of students after solving fraction problems. Interviews were conducted verbally with students to obtain information or explanations regarding their ability to solve problems. The validity of this research data uses triangulation of methods and sources. Method triangulation is done by verifying data from the same source using various methods. Meanwhile, source triangulation is done by re-examining data collected from various sources (Sutama et al., 2022).

The data analysis technique in this study uses the flow method, which is the process of analysing data through activities to collect or compile information systematically. The process begins with the results of observations, documentation, and interviews. Then, the data is organised, analysed, synthesised, arranged, and concluded so that it can be clearly understood. The data were analysed based on the test scoring guidelines presented in Table 1.

Table 1. Test Result Assessment Criteria

Value	Criteria
$x \geq 80$	High
$65 < x > 80$	Medium
$x \leq 65$	Low

Source: (Hilyani et al., 2020)

The score scoring formula for the test on fraction material is as follows:

$$x = a \times 4 \quad (1)$$

Description:

$x =$ Student's correct answer score

$a =$ Student's correct answer

$4 =$ Number of indicators of Polya's problem-solving ability

Results and Discussion

The study found variations in students' problem-solving abilities. Figure 1 displays the distribution of students in each problem-solving ability category. The problem-solving ability of

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students in the high category is 75%, the medium category is 15%, and the low category is 10%. Then, three student answers were taken. Namely, S-1 experienced high problem-solving ability, S-2 experienced moderate problem-solving ability, and S-3 experienced low problem-solving ability. Furthermore, students' problem-solving abilities will be studied using Polya's step indicators based on the high, medium and low ability categories.

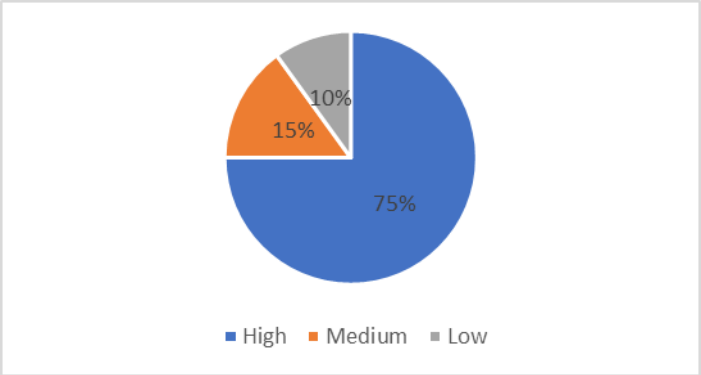

Figure 1. Percentage of Students in Each Category

Students with High Problem Solving Ability

Students with high problem-solving ability are characterised by achieving all problem-solving indicators using Polya's steps. This can be seen from the results of student work that can solve problems correctly according to Polya's steps and fulfil all four indicators of mathematical problem-solving ability. Figure 2 shows S-1's answer when working on fraction problems.

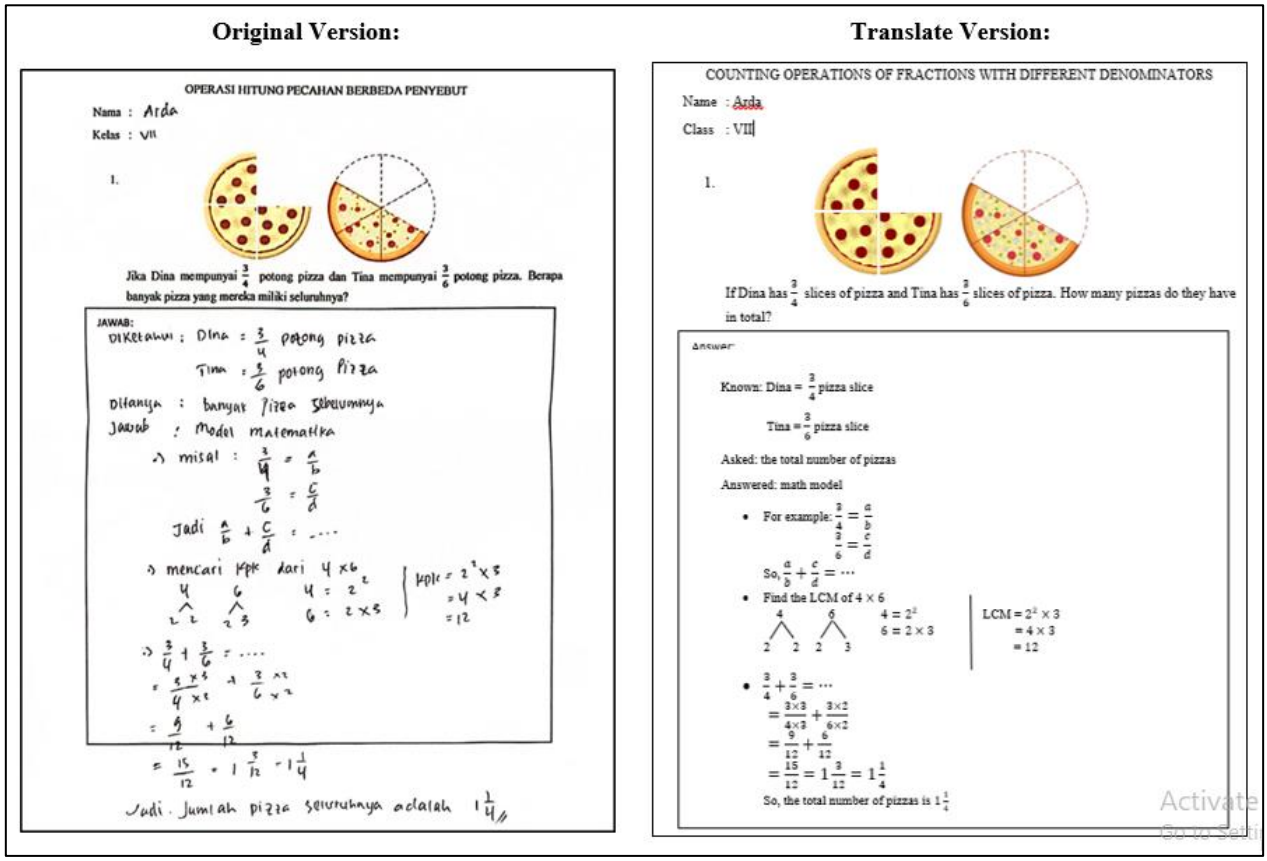

Figure 2. Results of S-1 Answers of High Problem-Solving Ability

From Figure 2, S-1 can understand the problem correctly, can plan the solution, complete the solution plan appropriately, and re-check the answer to get the correct answer. Overall, using Polya's

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steps, S-1 solved the problem by fulfilling all problem-solving indicators. On his answer sheet, S-1 can show the known and questionable information in the problem and solve the problem correctly. Based on the interview with S-1, the subject understood the meaning of the problem and explained the steps for solving it correctly. This is in line with the research findings of Purnomo et al. (2024), which show that students with high ability can solve fraction problems correctly.

Students understand the problem by displaying known, asked, and answered questions, and they can plan problem-solving, marked by writing a mathematical model and determining the KPK or the same denominator. Students can carry out the solution plan marked by performing the fraction addition operation appropriately. At the end of the solution, students double-check the answer marked by writing the conclusion on the answer sheet. Thus, S-1 is categorised as having high problem-solving ability because it can fulfil all problem-solving indicators by using Polya's steps appropriately.

Students with Medium Problem Solving Ability

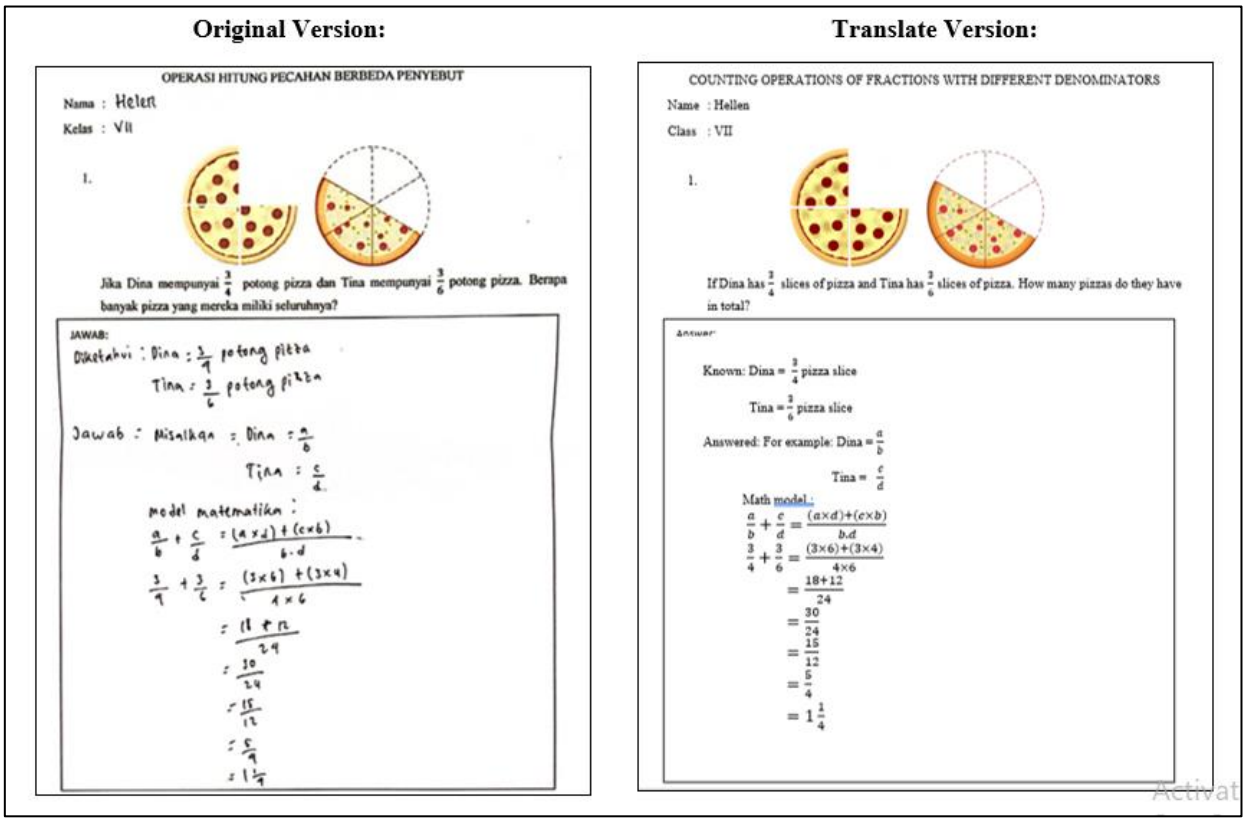

Figure 3. Results of S-2 Answers of Medium Problem-Solving Ability

Figure 3 shows that S-2 solved the problem well, although he did not fulfil all the indicators of problem-solving using Polya's steps. Based on the answer sheet, S-2 can understand the problem. Still, it is not optimal because he only records the known information without mentioning what is asked in the problem, does not include the strategy that will be used, and does not recheck the results of his answer. Furthermore, S-2 was able to implement the solution plan appropriately.

During the interview, S-2 explained that the subject needed to remember to write what the question asked because S-2 was in a hurry to do it. In addition, S-2 also found it challenging to implement the solution plan. To overcome this, S-2 read the problem repeatedly and then solved it correctly. However, he could not simplify it into the simplest form, so S-2 rechecked his answer. S-2 was able to understand the problem and explain the solution process. In accordance with the research of Pardiansyah et al. (2021), shows that students with moderate problem-solving ability are good enough at solving problems and fulfil all indicators, but still need to be more optimal at the problem-

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understanding stage. Therefore, the ability to solve problems using Polya's steps is good enough, but still requires improvement, especially at the stage of understanding the problem.

Students can understand the problem by displaying what is known and answered. However, it is not optimal, because it needs to write what is asked about the problem. S-2 can plan problem-solving, which is characterised by writing a mathematical model, determining the KPK or common denominator. Students can carry out the solution plan marked by performing the fraction addition operation appropriately, then at the end of the solution, students double-check the answer marked by writing the conclusion on the answer sheet. Thus, S-2 is categorised as having moderate problem-solving ability because it fulfils all problem-solving indicators using Polya's steps but not maximally at the stage of understanding the problem (Pratikno & Retnowati, 2018).

Students with low problem-solving ability

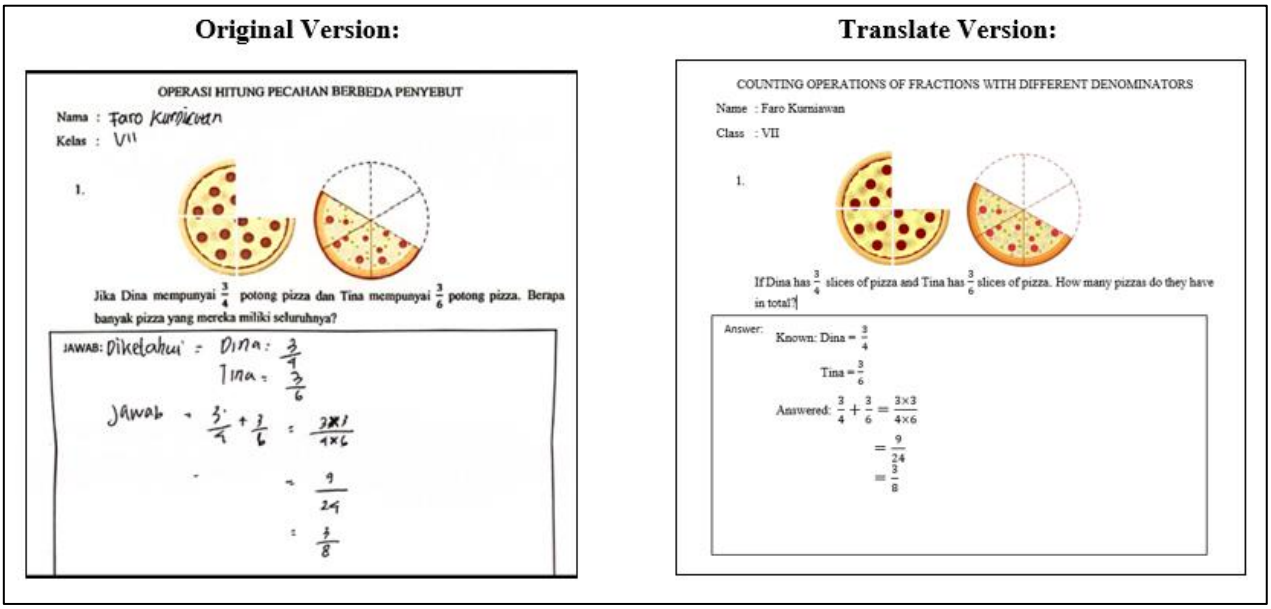

Figure 4. Results of S-2 Answers of Low Problem-Solving Ability

Students with low problem-solving ability are characterised by the fact that S-3 cannot fulfil the indicators of Polya's steps. Based on S-3's answer sheet in Figure 3, the results of S-3's solution could understand the problem and write what was known from the problem, but did not write what was asked. The next step was planning the solution. S-3 did not write the solution plan used to solve the problem. S-3 immediately solved the problem by operating the fractions given in the problem. However, it was still inaccurate because it did not use the concept of fractions with different denominators and S-3 did not provide conclusions about the results obtained. Based on the results of interviews conducted with S-3, students do not understand what Polya's steps are in solving fraction problems and still have difficulty solving the concept of fractions with different denominators. Thus, the solution results found by S-3 are still not correct. In accordance with research by Patmala & Erita (2024), it states that low category students need help in applying the concept of problem-solving.

The theoretical implication is that researchers explore the understanding of how students solve math problems. Simply put, the theory of mathematical problem solving is a theory that involves structured steps, where Polya's steps become one of the most widely used approaches of the many steps that exist. Applying Polya's steps, students will have a more transparent and more systematic framework for dealing with and solving problems, which can develop students' problem-solving skills more effectively. The practical implication is that students can develop their potential according to their problem-solving abilities. In mathematics learning, teachers can implement differentiated learning by considering students' differences in problem-solving ability.

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Conclusion

Based on the results and discussion, it can be concluded that most students showed high problem solving ability in HLT-based RME learning, with 75% of students successfully meeting the problem solving indicators. A total of 15% of students with moderate ability did not meet the indicators at the stage of understanding the problem and planning the solution, while 10% of students with low ability only met some indicators. This research provides an opportunity to explore the factors that influence the success of HLT-based RME model implementation, such as the influence of student characteristics and comparison with other learning models.

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