



PRE-SERVICE MATHEMATICS TEACHER'S ABILITY IN DESIGNING MATHEMATICAL TASK: AN ANALYSIS OF LESSON PLANS

Sofyan Mahfudy^{1*}, Mauliddin^{1,2}

¹Prodi Tadris Matematika, FTK, UIN Mataram, Mataram, NTB, 83116, Indonesia

²Prodi Ilmu Aktuaria, FMIPA, Universitas Hasanuddin, Makassar, Sulawesi Selatan, 90245, Indonesia

e-mail: ¹*sofyan_mahfudy@uinmataram.ac.id, ²mauliddin@unhas.ac.id

*Corresponding Author

Received: 25-07-2024; Revised: 15-08-2024; Accepted: 30-08-2024

Abstract: Mathematical tasks have an essential role in the mathematics learning process. Well-designed mathematical tasks will provide a learning opportunity for students to develop their mathematical ideas. The ability of pre-service mathematics teachers to design mathematical tasks during their academic studies significantly influences how they teach in actual learning. One of the ways to investigate pre-service mathematics teachers' ability to design tasks is by analyzing the tasks in the lesson plans they proposed. This study aims to scrutinize the level of cognitive demand for mathematics tasks designed by pre-service mathematics teachers in their lesson plans to get a thorough understanding of their ability to design tasks. This descriptive qualitative research analyzed mathematical tasks in 25 lesson plans proposed by pre-service mathematics teachers using the framework developed by Stein and Smith (1998). The results of this study show that most pre-service teachers designed tasks with a lower cognitive demand classification with types of memorization and procedures without connections. The mathematical tasks designed by pre-service mathematics teachers also have yet to lead to argumentation and the development of mathematical ideas through exploration or doing mathematics. Thus, based on the findings in this study, designing mathematics tasks with the potential to promote students' mathematical understanding remains a challenge for pre-service mathematics teachers. This article discusses and presents the implications of research results and recommendations for designing mathematical tasks.

Keywords: mathematical task; cognitive demand; pre-service mathematics teachers; lesson plans

How to Cite: Mahfudy, S., Mauliddin. (2024). Pre-Service Mathematics Teacher's Ability in Designing Mathematical Task: An Analysis of Lesson Plans. *JP2M: Jurnal Pendidikan dan Pembelajaran Matematika*, Vol.10 No.2.,(334-341) <https://doi.org/10.29100/jp2m.v10i2.6525>



Introduction

Mathematical tasks are essential for mathematics teaching and learning (Sullivan et al., 2012) and highly influence students' thinking processes, level of engagement, and achievement of learning outcomes (Breen & O'Shea, 2018). In the learning process, what students learn is strongly determined by the mathematical tasks provided by the teacher (Grevholm et al., 2009; Henningsen & Stein, 1997; Paolucci & Wessels, 2017). Breen & O'Shea (2018) argued that mathematical tasks are tools to initiate mathematical activities in the learning process. Therefore, well-designed mathematical tasks will impact the effectiveness of learning.

Sullivan et al. (2012) define mathematical tasks as information that serves as the first step that promotes student learning, presented to students in the form of questions, situations, and instructions



and being a starting point and context for learning. Meanwhile, Stein & Smith (1998) stated that mathematical tasks are one segment of classroom activities for developing a mathematical idea. Mathematical tasks can include several relevant problems or extended work on one complex problem. Activity refers to the thoughts and actions, physical, oral, written, and recorded, that students perform in response to a given task (Sullivan et al., 2012). Specifically, Henningsen & Stein (1997) described a mathematical task as a set of problems or a complex problem that focuses students' attention on a mathematical idea. Based on the definitions, this study defines mathematical tasks as a set of problems designed by the teacher as a student activity to develop an understanding of mathematical ideas.

Sullivan et al. (2012) suggest some characteristics of mathematical tasks that consist of (1) providing an appropriate context and level of difficulty, (2) stimulating the growth of cognitive networks, thinking, creativity, and reflection, and (3) explicitly covering deep mathematical content. Meanwhile, Anthony & Walshaw (2007) mentioned three main things that characterize mathematical tasks, i.e.: (1) mathematical tasks should activate students in mathematical thinking, (2) mathematical tasks should consider the knowledge and experience that students already have, and (3) mathematical tasks should utilize 'mathematical tools' to build students' understanding of concepts.

The ability of in-service and pre-service mathematics teachers (PSMTs) to design mathematics tasks is required to achieve effective and meaningful mathematics learning. Generally, the PSMTs have not yet taken on the actual role of a teacher in the classroom, but they are a potential component that will determine the quality of mathematics education in the future. Their education aims not merely to acquire knowledge and skills about teaching and learning but as an initial stage in being a real teacher (Wenger, 1999). PSMTs who have completed various learning design courses already have knowledge and experience in designing lesson plans, including the tasks to help students understand mathematical ideas. However, the ability to design mathematics tasks for pre-service mathematics teachers deserves attention from mathematics educators. Based on the author's experience during microteaching courses in the mathematics education department, a glance analysis at the tasks designed by pre-service mathematics teachers showed that the tasks tend to measure the students' understanding rather than help them to promote their understanding. Therefore, a thorough analysis of these mathematics tasks will provide us with a deep understanding of pre-service mathematics teachers' ability to design tasks.

Some previous studies have examined PSMTs' ability to design mathematical tasks in lesson plans they proposed (Anhalt et al., 2006; Sapkota, 2022; Oliveira & Henriques, 2021) and the tasks provided by school mathematics textbooks (Özgeldi & Esen, 2010; Wijaya et al., 2015; Jones & Tarr, 2007; Yang et al., 2017; Charalambous et al., 2010; Yang, 2017). Dogan (2020) also evaluated teachers' competencies in designing authentic mathematical modelling tasks. In the Indonesian context, few studies have examined PSMTs' competence in designing mathematical tasks. However, investigating pre-service teachers' ability to design mathematical tasks proposed in their lesson plans remains interesting. One of these issues is cognitive demands on mathematical tasks as an integral learning dimension (Tekkumru-Kisa et al., 2015). This study analyses mathematics tasks designed by PSMTs based on the cognitive demands developed by Stein & Smith (1998). Analyzing the tasks on the lesson plans will reveal the pre-service mathematics teacher's ability to design mathematical tasks.

Method

This study used document analysis in a descriptive qualitative paradigm (Cohen et al., 2018; Creswell, 2014). The unit of analysis was the mathematical tasks that PSMTs designed in their proposed lesson plans. The tasks were coded by applying criteria synthesized from theories and frameworks related to the cognitive demands of mathematical tasks (Stein & Smith, 1998).

The data in this study were collected from lesson plans proposed by PSMTs. The participants were 25 selected PSMTs in mathematics teacher education programs in Indonesia. All participants were in the third year of the teacher education programs and enrolled in the MicroTeaching course. The

MicroTeaching course is designed to prepare PSMTs to have competence in preparing and implementing teaching and learning in small classes. In this course, all participants were assigned different mathematics topics at the junior and senior high school levels. Furthermore, all pre-service teachers were required to design a lesson plan based on the material they received. The lesson plan will be used to guide their teaching practice. The structure of lesson plans designed by PSMTs refers to the Learning Trajectory theory proposed by Simon (1995), which contains three components: learning goals, learning activities, and hypothetical learning process. In particular, mathematical tasks are part of the activities to create effective and meaningful mathematics learning. Clements & Sarama (2004) stated that mathematical tasks are designed to elicit mental processes or actions that are predicted to help students achieve developmental levels of thinking. In this study, the lesson plan was structured by adopting the Learning Trajectory components proposed by Simon (1995), which include: starting point of learning, learning goals, learning activities, mathematical tasks, and hypothetical learning process. PSMTs could ask questions during the four-course sessions and get feedback when designing proposed lesson plans. PSMTs revised their lesson plans 2 to 3 times during the four-course session. This study analyzed the mathematical tasks in the final or last revised lesson plans.

The tasks designed by pre-service mathematics teachers were analyzed using the classification of cognitive demands proposed by Stein & Smith (1998). There are four types of mathematics tasks: memorization, procedure without connection, procedure with connection tasks, and doing mathematics. The first two mathematical tasks (memorization and procedure without connection) are categorized as tasks with low-level cognitive demands. The last two mathematical tasks (procedure with connection and doing mathematics) are categorized with high-level cognitive demands. The codes with descriptions and examples of guiding questions for analyzing mathematical tasks are shown in Table 1.

Table 1. Criteria for the level of cognitive demands in mathematical tasks

Criteria for task	Description	The example of guiding questions
Memorization (M)	Tasks that only require rules, procedures, or concepts that the student has memorized to complete	<ul style="list-style-type: none"> ▪ Does the task involve reproducing facts, rules, formulas, or definitions that have already been learned? ▪ Whether the task cannot be completed by using the procedure? ▪ Can the task be completed in a short period? ▪ Is the reproduction required by students clearly and directly stated? ▪ Does the task have no connection to the concept underlying the fact, rule, formula, or definition being learned?
Procedure without connection (PWoC)	Tasks that involve procedures but do not require connections across rules or concepts to complete	<ul style="list-style-type: none"> ▪ Is the task algorithmic? ▪ Does the task utilize clear procedures as directed? ▪ Does the task require a limited cognitive level to complete successfully? ▪ Does the task focus on the correct answer rather than developing mathematical understanding? ▪ Does the task require no explanation? Alternatively, it requires explanation but only focuses on explaining the procedures used.
Procedure with connection (PC)	Tasks that require procedures and connections to complete	<ul style="list-style-type: none"> ▪ Does the task focus students' attention on using procedures to develop a deeper understanding? ▪ Does the task provide general instructions or procedures, and it's directly related to the conceptual idea? ▪ Is the task presented in a variety of ways? For example, diagrams, manipulatives, symbols, and context.

Criteria for task	Description	The example of guiding questions
Doing mathematics (DM)	Tasks that require complex cognitive processes to complete	<ul style="list-style-type: none"> ▪ To complete the task successfully, can the student follow a general procedure but through some reasoning or thinking? ▪ Does the student need to think about the conceptual ideas underlying a procedure required to complete the task successfully? ▪ Does the task require complex and non-algorithmic thinking? ▪ Does the task require students to explore and understand the characteristics of mathematical concepts, processes, or connections? ▪ Does the task require students to access relevant knowledge and experience and use it appropriately to solve? ▪ Does the task require higher cognitive effort and student curiosity due to the unpredictable way to solve it?

Results and Discussion

The results of this study revealed the types of cognitive demands included in the mathematics tasks designed by PSMTs in their lesson plans. First, this section summarizes the coding results of the mathematics tasks based on the classification criteria in Table 1. Secondly, the general results are presented as selected examples of mathematics tasks designed by pre-service teachers, along with their analysis and discussion. Finally, the discussion and implications of the research results for the learning process and mathematics teacher education programs are presented. Out of the 25 lesson plans proposed by the PSMTs, 25 mathematical tasks were analyzed. Based on the analysis, the number of mathematics tasks in each category of cognitive demands is shown in Table 2.

Table 2. The number of tasks in each cognitive demand category

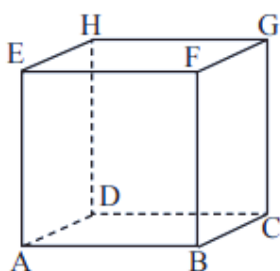
Lower-Level Cognitive Demand		Higher-Level Cognitive Demand	
M	PWoC	PC	DM
4 (16%)	17 (68%)	4 (16%)	0 (0%)

As shown in Table 2, out of the 25 tasks in the lesson plans analyzed, 21 (84%) tasks were of lower cognitive demand, and 4 (16%) tasks were of higher cognitive demand. Specifically, no ‘doing mathematic’ tasks are in the higher-level cognitive demand category. The following are some examples and analyses of tasks designed by PSMTs.

Memorization Task

Task 1 (Designed by EN)

Given a cube ABCD.EFGH as shown below.



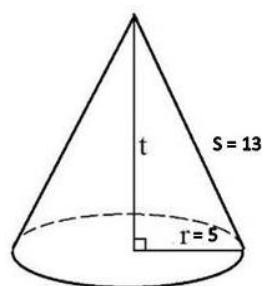
Based on the figure of the cube, answer the following questions:

- How many face diagonals are there in the cube? List all the face diagonals!
- How many body diagonals are there in the cube? Write down all the body diagonals!

PSMTs (EN) designed Task 1 for junior high school students on the topic of diagonals in cubes. The learning goal in the lesson plan that contains this task is to understand the concept of ‘face and body diagonal’ in a three-dimensional shape. This task requires students to determine how many face and body diagonals are in the given cube and list all the diagonals. These problems typically can be found in Indonesian mathematics textbooks in explanation or worked example sections. We categorized Task 1 as a lower-level cognitive demand task with a memorization type. To solve this problem, students did not require any specific procedures. Indeed, to answer this task correctly, it requires definitions and concepts related to diagonals in a cube, but students will most likely use their memories from previous learning experiences or materials. Moreover, cubes are a type of three-dimensional shape that teachers very often use to explain the three-dimensional topic in classroom explanation sessions. From the student's perspective, we predict that the intended solution to this task is quite clear and does not cause ambiguity. Task 1 can also be completed in a short period. Therefore, when students respond and grapple with this task, it is less of an opportunity to provide a learning experience and challenge for students.

Procedure Without Connection Task

Task 2 (Designed by S)



Based on the figure above, determine the volume of the cone.

Task 3 (Designed by SSZ)



The figure above shows a pyramid with a square base. The square has sides 230 m long, and the pyramid's altitude is 146 m. Determine the volume of the pyramid.

Tasks 2 and 3 above are tasks on the volume of cones and pyramids designed for junior high school students. Task 2 and Task 3 are from two lesson plans designed by two PSMTs (S and SSZ). Respectively, the learning goals are to find the formula for the volume of a cone and a pyramid. We categorized both tasks as lower-level cognitive demands with procedures without connection type. Without context, the information required to complete Task 2 is clear and easy for students to find without consideration and analysis. Regarding Task 3, although it uses a real-world context, all the information needed to answer the question correctly is explicitly accessible for students to find in the task. Therefore, these two tasks are algorithmic, where students only need to apply procedures that they were previously familiar with, such as the cone and pyramid volume formula. By first finding the altitude of the cone and the area of the base of the pyramid, students will be able to complete both tasks. With a single question asking students to determine the volume of a cone and a pyramid, both tasks focus on the correct answer rather than developing students' mathematical understanding.

Procedure with connection Task

Task 4 (Designed by M)

A group of 68 people has the plan to go traveling. They will rent two types of cars. Based on information from the rental, car A can carry four passengers, while car B can carry six passengers. The total number of cars to

be rented is at least 12, and car B will be rented at least 8. If the rental price of car A is IDR 450,000 and the rental price of car B is IDR 500,000, then determine the number of each type of car that should be rented so the costs will be minimum!

The topic of Task 4 is program linear, and it was designed (by M) for high school students. The learning goal in the lesson plan containing this task is to determine the optimum value using the graphical method. This task is a word problem in the context of a real-world problem related to optimizing the cost of travelling. Referring to the task analysis guide, We categorized this task as a higher-level cognitive demand with type a procedure with connection task. Students must follow a specific procedure they cannot directly discover to complete this task. In other words, the procedure to be applied must be based on cognitive thinking and judgment of the information provided by the task. For instance, how students correctly determine the constraint and objective functions of the problem on the task is a challenge. Mathematizing a word problem into a formal mathematical model is difficult for students. In this case, students must consider the types of operations and symbols appropriate to the word problem. In other words, students need more cognitive level effort to perform the steps of solving the task.

The findings of this study show that all PSMTs have successfully designed mathematics tasks as part of mathematics learning activities. Nevertheless, the results and data analysis have shown that designing an effective mathematical task for learning is difficult for PSMTs. Most tasks designed by PSMTs were categorized as low-level cognitive demand tasks with memorization and procedure without connection types. Based on previous studies, these kinds of tasks lead to one type of opportunity for students thinking (Stein & Smith, 1998). To address this task, the student merely needs to reproduce the procedure that he/she already knows, thus not developing mathematical understanding. In contrast, mathematical tasks with high-level cognitive demands, especially tasks of the doing-mathematics type, will provide opportunities for students thinking. This type of task allows students to explore and understand the characteristics of mathematical ideas, concepts, processes, or relationships. Hence, teachers should improve their knowledge and skills in developing high-level cognitive demand tasks, especially tasks with type doing-mathematic.

Several previous studies have shown that tasks set up by teachers in the classroom are not always implemented as intended. The teacher may create a task as a high-level cognitive demand, but in its implementation, students' responses and work on the task do not match what the teacher expects. A task that had been set up at high-level cognitive demand but implemented at a much low-level cognitive demand. Therefore, in addition to having the competence to design tasks with high-level cognitive demand, a PSMTs must recognize the factors that can maintain a task so that this task works as expected. Stein & Smith (1998) proposed some related factors that can maintain high-level cognitive demand tasks are (1) providing scaffolding for students' thinking and reasoning, (2) emphasizing students' justifications, explanations, and meaningfulness through questions, comments, and feedback, (3) considering students' prior knowledge, and (4) providing sufficient time to allow students to explore.

Based on the results of this study, pre-service and even in-service teachers should continuously explore various problems to design a well-designed mathematics task. They must create a group of discussions with their colleagues, discussing and reflecting on their learning, especially designing effective mathematics tasks. Therefore, several considerations in designing tasks that make use of tools, including epistemological and mathematical, tool-representational, pedagogical, and discursive considerations, should be recognised by PSMTs (Leung & Bolite-Frant, 2015). PSMTs should be aware that the mathematical tasks they set up for their students in the classroom are an essential aspect of learning and greatly influence the learning opportunities of students (Sullivan et al., 2012). Well-designed tasks with high-level cognitive demands are most likely to help students develop the mathematical ideas they are learning and promote interaction in the classroom (Christiansen & Walther, 1986). Mathematics teacher education and training programs must ensure that all pre-service teachers

are competent in designing mathematics tasks. It is worth considering that mathematics teacher education programs have a particular course that focuses on providing pre-service teachers with the competence to design mathematics tasks.

Conclusion

This study aimed to investigate pre-service teachers' competence in designing mathematics tasks. Generally, pre-service mathematics teachers do not have sufficient knowledge and experience to design mathematics tasks categorized as high-level cognitive demand tasks. In designing mathematics tasks, pre-service teachers tend to adopt, even take for granted, the tasks found in mathematics textbooks without cognitive study or consideration. Designing mathematics tasks with high-level cognitive demand, especially the doing-mathematics type, remains challenging for pre-service and in-service teachers. Good skills in designing mathematics tasks will help teachers conduct mathematics learning that encourages argumentation and the development of mathematical ideas so that learning objectives can be achieved well. However, this study only focused on teachers' ability to design mathematical tasks through evaluation and reflection on their lesson plans. This study did not look at how the mathematical tasks were applied in actual teaching and learning. Another limitation of this study is the analysis of mathematical tasks carried out by a single coder. It certainly creates a bias that may affect the study results.

Acknowledgement

The authors thank the Lembaga Penelitian dan Pengabdian Masyarakat (LP2M) Universitas Islam Negeri Mataram for funding this research and the students of the Tadris Matematika, Fakultas Tarbiyah, and Keguruan for their participation in this study.

References

- Anhalt, C. O., Ward, R. A., & Vinson, K. D. (2006). Teacher candidates' growth in designing mathematical tasks as exhibited in their lesson planning. *Teacher Educator*, 41(3), 172–186. <https://doi.org/10.1080/08878730609555382>
- Anthony, G., & Walshaw, M. (2007). *Effective Pedagogy in Mathematics/Pāngarau: Best Evidence Synthesis Iteration (BES)*. Wellington: Ministry of Education. https://thehub.sia.govt.nz/assets/documents/42433_BES_Maths07_Complete_0.pdf
- Breen, S., & O'Shea, A. (2018). Designing Mathematical Thinking Tasks. *PRIMUS*, 29(1), 9–20. <https://doi.org/10.1080/10511970.2017.1396567>
- Charalambous, C. Y., Delaney, S., Hsu, H. Y., & Mesa, V. (2010). A comparative analysis of the addition and subtraction of fractions in textbooks from three Countries. *Mathematical Thinking and Learning*, 12(2), 117–151. <https://doi.org/10.1080/10986060903460070>
- Christiansen, B., & Walther, G. (1986). Task and activity. In *Perspectives on mathematics education* (pp. 243–307). Dordrecht: Springer Netherlands.
- Clements, D. H., & Sarama, J. (2004). Learning Trajectories in Mathematics Education. *Mathematical Thinking and Learning*, 6(2), 81–89. https://doi.org/10.1207/s15327833mtl0602_1
- Cohen, L., Manion, L., & Morrison, K. (2018). *Research methods in education* (8th ed.). Milton Park: Routledge.
- Creswell, J. W. (2014). *Research design: Qualitative, quantitative, and mixed methods approaches* (4th ed.). Thousand Oaks, California: Sage.
- Dogan, M. F. (2020). Evaluating Pre-Service Teachers' Design of Mathematical Modelling Tasks. *International Journal of Innovation in Science and Mathematics Education*, 28(1), 44–59.

- Grevholm, B., Millman, R., & Clarke, B. (2009). Function, form and focus: The role of tasks in elementary mathematics teacher education. In B. Clarke, B. Grevholm, & R. Millman (Eds.), *Tasks in primary mathematics teacher education* (Vol. 4, pp. 1–5). Springer.
- Henningsen, M., & Stein, M. K. (1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education*, 28(5), 524–549. <https://doi.org/10.5951/jresmetheduc.28.5.0524>
- Jones, D. L., & Tarr, J. E. (2007). Examination of the levels of Cognitive Demand Required by Probability Tasks in Middle Grades Mathematics Textbooks. *Statistics Education Research Journal*, 6(2), 4–27. <http://www.stat.auckland.ac.nz/serj>
- Leung, A., & Bolite-Frant, J. (2015). Designing Mathematics Tasks: The Role of Tools. In A. Watson & M. Ohtani (Eds.), *New ICMI Study 22* (2nd ed., pp. 191–225). Springer. <http://www.mathunion.org/ICMI/>
- Oliveira, H., & Henriques, A. (2021). Preservice Mathematics Teachers' Knowledge about the Potential of Tasks to Promote Students' Mathematical Reasoning. *International Journal of Research in Education and Science*, 7(4), 1300–1319. <https://doi.org/10.46328/ijres.2472>
- Özgeldi, M., & Esen, Y. (2010). Analysis of mathematical tasks in Turkish elementary school mathematics textbooks. *Procedia - Social and Behavioral Sciences*, 2(2), 2277–2281. <https://doi.org/10.1016/j.sbspro.2010.03.322>
- Paolucci, C., & Wessels, H. (2017). An examination of pre-service teachers' capacity to create mathematical modeling problems for children. *Journal of Teacher Education*, 68(3), 330–344.
- Sapkota, B. (2022). Preservice Teachers' Conceptualizations of Mathematical Tasks. *The Mathematics Educator*, 30(2), 3–32.
- Simon, M. A. (1995). Reconstructing Mathematics Pedagogy from a Constructivist Perspective. *Journal for Research in Mathematics Education*, 26(2), 114–145. <https://doi.org/10.5951/jresmetheduc.26.2.0114>
- Stein, M. K., & Smith, M. S. (1998). Mathematical tasks as a framework for reflection: From research to practice. *Mathematics Teaching in the Middle School*, 3(4), 268–275.
- Sullivan, P., Clarke, D., & Clarke, B. (2012). *Teaching with Tasks for Effective Mathematics Learning* (Vol. 9). New York: Springer Science & Business Media. <https://doi.org/10.1007/978-1-4614-4681-1>
- Tekkumru-Kisa, M., Stein, M. K., & Schunn, C. (2015). A framework for analyzing cognitive demand and content-practices integration: Task analysis guide in science. *Journal of Research in Science Teaching*, 52(5), 659–685. <https://doi.org/10.1002/tea.21208>
- Wenger, E. (1999). *Communities of practice: Learning, meaning, and identity*. Cambridge University Press.
- Wijaya, A., Heuvel-Panhuizen, M., & Doorman, M. (2015). Opportunity-to-learn context-based tasks provided by mathematics textbooks. *Educational Studies in Mathematics*, 89, 41–65. <https://doi.org/10.1007/s10649-015-9595-1>
- Yang, D. C. (2017). Study of fractions in elementary mathematics textbooks from Finland and Taiwan. *Educational Studies*, 44(2), 1–22. <https://doi.org/10.1080/03055698.2017.1347493>
- Yang, D. C., Tseng, Y. K., & Wang, T. L. (2017). A comparison of geometry problems in middle-grade mathematics textbooks from Taiwan, Singapore, Finland, and the United States. *Eurasia Journal of Mathematics, Science and Technology Education*, 13(7), 2841–2857. <https://doi.org/10.12973/eurasia.2017.00721a>