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OPTIMAL CONTROL ANALYSIS OF A SEIV EPIDEMIC MODEL WITH VACCINATION AND EDUCATION

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ABSTRACT
This paper discusses optimal control of a mathematical epidemic model governed by an ODE system with saturated incidence rate. An epidemic model is developed using optimal control theory by dividing the population into Susceptible, Exposed, Infected, and Vaccinated (SEIV) sub populations. In the model we assume that half of new born individual have been vaccinated. Optimal control is conducted by adding two control variables namely vaccination and education. The aim of optimal control is to minimize the density of exposed subpopulation, infected subpopulation, and the cost of control. Optimal control is obtained by applying Pontryagin minimum principle. Furthermore, the optimal control problem is solved numerically by using Forward-Backward Sweep method. Three approaches were used to conduct numerical simulations, applying vaccination control without education, applying education control without vaccination, and using both vaccination and education control. There are Numerical simulations show that vaccination and education are effective in reducing exposed and infected subpopulation.

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I. INTRODUCTION

The dynamics of epidemic disease are frequently described by mathematical population modeling. A model of the SEIV epidemic including vaccination and vertical transmission was examined in [1]. Their findings were expressed as a fundamental reproduction number (R_0) and they performed a bifurcation analysis to determine the prerequisities for the system to display backward bifurcation. He reviewed a vaccination model in [2]that had a nonlinear incidence rate and a vaccination waning period similar to [3], but the model was numerically simulated to observe how the variable rate varied. A saturated incidence rate $g(I)$ was include to epidemic model in [4]. The work of [3] is expanded upon in [5] by using a saturated incidence rate as used by [4].

Improving control and eventually eradicating the infection from the population are two main goals of research on infectious diseases. In this approach, models can be an effective tool that help us target control measures more precisely or make the best use of scarce resources. There are various types of control measures that work by lowering the average rate of transmission between susceptible and infectious individuals. Which control strategy is used will depend on the disease, the hosts, and the scale of the epidemic. Strategies such as treatment, vaccination, and educational campaigns can be employed in the context of epidemic diseases [6].

Optimal control theory is an additional field of mathematics that has substantial application in managing the transmission of infectious diseases. It is a powerful mathematical tool that can be used to make decisions involving complex biological situations [7]. It is frequently employed in preventing the spread of the majority of illnesses for which a vaccine or treatment is available. For example, [8] use optimal control theory to find the most effective control strategy to minimize the number of individuals who become infected in the course of an epidemic using both treatment and vaccination as control measures. In [7], they applied optimal control theory to reduce infected individuals by applying campaign education control. The educational control campaign is expected to change individual behaviour. As in [9] they also applied optimal control to reduce the spread of rotavirus infection by using three controls, namely vaccination, treatment, and education.

In this work, we added vaccination and education controls to the SEIV epidemic model [5]. An optimal control strategy is performed to reduce the exposed and infected subpopulations. Applying the Pontryagin minimum principle yields optimal control. The forward-backward sweep method is used to numerically solve the optimal control issue.

II. METHODS

A. Mathematical Model

Based on [5]we have a mathematical model that is divided into four subpopulations, namely Susceptible (S), Exposed (E), Infected (I), and Vaccinated (V). In this study, the SEIV epidemic model [5] is formulated by adding control variables vaccination (u_1) and education (u_2). It is advised that people get vaccinated to lower their risk of infection and its subsequent spread, as vaccinations can effectively prevent infection and its transmission. Furthermore, it is believed that education would help the population adopt better habits, such as upholding hygiene and cleanliness. The system of optimal control is defined as follows

$$\begin{aligned} S' &= (1-p)\pi - \frac{(1-u_2)\beta SI}{(1+\alpha I)} - \mu S + \omega V - u_1 S, \\ E' &= \frac{(1-u_2)\beta SI}{(1+\alpha I)} - (\mu+\sigma)E, \\ I' &= \sigma E - (\mu+\gamma)I, \\ V' &= p\pi - \mu V + \gamma I - \omega V + u_1 S \end{aligned} \tag{1}$$

Where p is the fraction of recruited individuals who are vaccinated. π is the recruitment of individuals that includes new born and immigrants into susceptible population. β is the rate at which susceptible individuals become infected by those who are infectious. σ is the rate at which exposed individuals become infectious. μ is the natural death rate. γ is the rate at which infected individuals are treated or recovered. ω is the rate at which vaccine wanes.

B. Optimal Control Problem

In this work, the main purpose is to minimize the function given by,

$$J(u_1,u_2) = \int_0^{t_1} (E + I + u_1^2 + u_2^2)dt \tag{2}$$

subject to the control system (1). The vaccination and the education cost weight in the objective function (2) are both 1. The system of equations in this instance states that the controls, u_1 dan u_2 will be minimized with a constraint function (1). The cost function is a nonlinear function that represents the cost-effectiveness of vaccination and education as two quadratic functions, ${u_1}^2$ dan ${u_2}^2$. Furthermore, based on equation (2), ${u_1}^*$and ${u_2}^*$ will be determined, resulting in the objective function's minimum value, which is

$$J(u_1^*, u_2^*) = \min_{(u_1,u_2)\in U} J(u_1,u_2)$$

Where $U = \{(u_1,u_2) : u_{1\min} \le u_1 \le u_{1\max}, u_{2\min} \le u_2 \le u_{2\max}\}$.

Based on objective function (2) and constrain function (1), we defined our Hamiltonian function for our control problem as:

$$
\begin{aligned}
H &= E + I + u_1^2 + u_2^2 \\
&+\lambda_1((1-p)\pi - \frac{(1-u_2)\beta SI}{(1+\alpha I)} - \mu S + \omega V - u_1 S) \\
&+\lambda_2(\frac{(1-u_2)\beta SI}{(1+\alpha I)} - (\mu+\sigma)E) \\
&+\lambda_3(\sigma E - (\mu+\gamma)I) \\
&+\lambda_4(p\pi - \mu V + \gamma I - \omega V + u_1 S)
\end{aligned} \tag{3}
$$

where λ_i for $i = 1, 2, 3, 4$ denote the adjoint variables associated to the state variables S, E, I, and V. By applying the appropriate partial derivatives of H with respect to the state variables, the system of adjoint equations can be constructed, as follows

$$
\begin{aligned}
\lambda_1' &= -\frac{\partial H}{\partial S} = \lambda_1(\frac{(1-u_2)\beta SI}{(1+\alpha I)} + \mu + u_1) - \lambda_2 \frac{(1-u_2)\beta SI}{(1+\alpha I)} - \lambda_4 u_1, \\
\lambda_2' &= -\frac{\partial H}{\partial E} = -1 + \lambda_2(\mu+\sigma) - \lambda_3\sigma, \\
\lambda_3' &= -\frac{\partial H}{\partial I} = -1 + \lambda_1 \frac{(1-u_2)\beta S}{(1+\alpha I)^2} - \lambda_2 \frac{(1-u_2)\beta S}{(1+\alpha I)^2} + \lambda_3(\mu+\gamma) - \lambda_4\gamma, \\
\lambda_4' &= -\frac{\partial H}{\partial V} = -\lambda_1\omega + \lambda_4(\mu+\lambda_4).
\end{aligned} \tag{4}
$$

By differentiating the Hamiltonian function with respect to each variable control, we get the stationary conditions:

$$
u_1^* = \frac{(\lambda_1 - \lambda_4)S}{2} \tag{5}
$$

and

$$
u_2^* = \frac{(\lambda_2 - \lambda_1)\beta SI}{2(1+\alpha I)} \tag{6}
$$

Control variables of a SEIV epidemic model with vaccination and education are defined in $u_{1min} \leq u_1 \leq u_{1max}$ and $u_{2min} \leq u_2 \leq u_{2max}$. So, we get

$$
u_1^*(t) = min\left[u_{1\max}, max\left[u_{1\min}, \frac{(\lambda_1 - \lambda_4)S}{2}\right]\right] \tag{7}
$$

and

$$
u_2^*(t) = min\left[u_{2\max}, max\left[u_{2\min}, \frac{(\lambda_2 - \lambda_1)\beta SI}{2(1+\alpha I)}\right]\right] \tag{8}
$$

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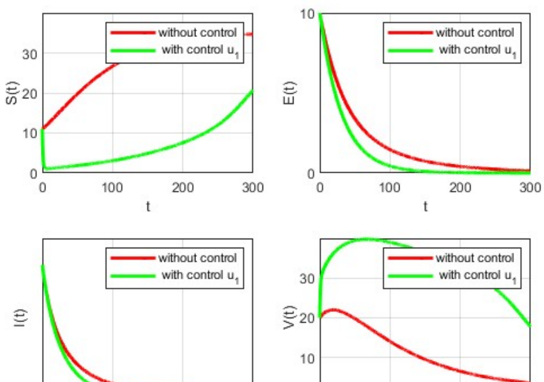
III. NUMERICAL RESULTS AND DISCUSSION

We employ the Forward-Backward Sweep approach to solve the optimal control problem. We use the parameter values given in Table 1 and initial values $S(0) = 11, E(0) = 10,\ I(0) = 5, V(0) = 20$.

TABLE I
PARAMETER VALUES

Symbol	Description	Value
p	the fraction of recruited individuals who are vaccinated	0.25
π	recruitment of individuals that includes new born and immigrants into susceptible population.	0.1
β	rate at which susceptible individuals become infected by those who are infectious	0.002
σ	rate at which exposed individuals become infectious	0.03
μ	natural death rate	0.003
γ	rate at which infected individuals are treated or recovered	0.1
ω	rate at which vaccine wanes	0.01

The solution of model (1) with three strategies is shown in Figure 1, Figure 2, and Figure 3. Figure 1 describes the solution of model (1) with and without vaccination. The solution of the model with and without education is shown in Figure 2. Figure 3 illustrates the solution of the model by applying a combination of control vaccination and education. Based on Figure 1, Figure 2, and Figure 3, we can see that all strategies reduce the number of exposed and infected subpopulation significantly.



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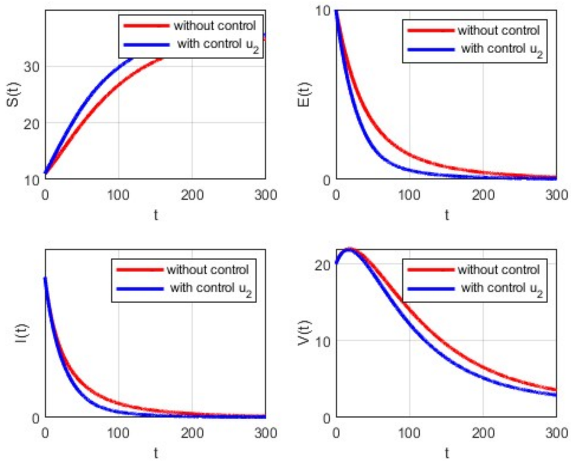

Figure 2. Simulation result of the model (1) with and without education

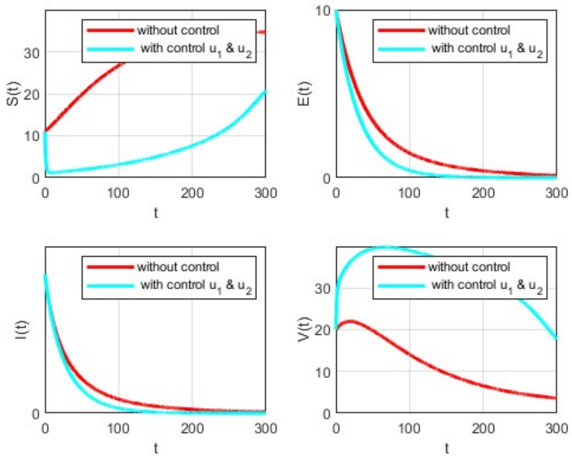

Figure 3. Simulation result of the model (1) with two controls

The optimal control profiles for each strategy is depicted in Figure 4, Figure 5, and Figure 6.

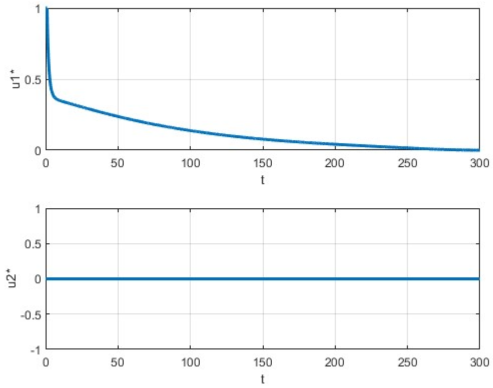

Figure 4.. The optimal control profile of u_1

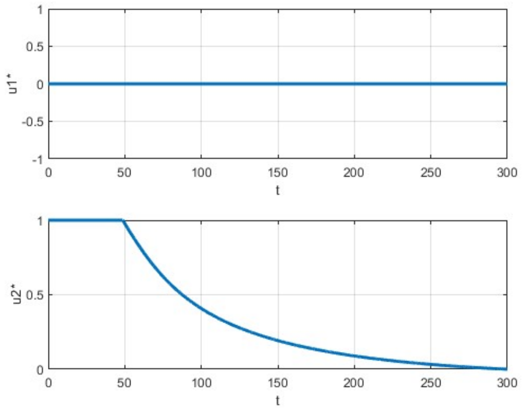

Figure 5.. The optimal control profile of u_2

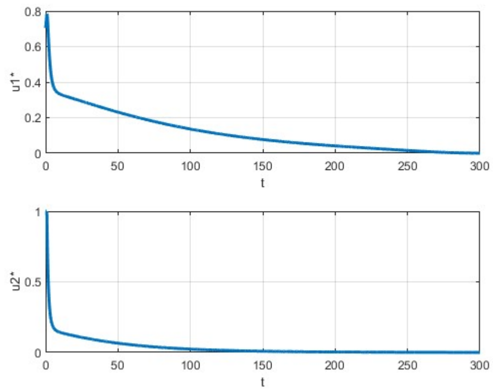

Figure 5.. The optimal control profile of u_1 and u_2

TABLE 2

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THE COST OF EACH CONTROL STRATEGY

Startegy	Cost
u_1	468.0449
u_2	551.9912
u_1 and u_2	465.5194

Based on the simulation result, we can conclude that by using w1 = 1,w2 = 1 as the weights, the combination of control u_1 and u_2 is effective to reduce the number of infected subpopulation with minimum cost of control.

IV. CONCLUSION

In this study, optimal control of a SEIV epidemic model has been carried out. Two controls were used in this model, namely vaccination and education. By using Pontryagin Minimum Principle, we obtained the optimal control of the model. Based on the results of the study, the provision of these two controls has a positive impact as it can reduce the number of infected subpopulations.

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